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The Non-Abelian Born-Infeld Action and Noncommutative gauge theory

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Abstract

In this paper we explicitly show the equivalence between the non-Abelian Born-Infeld action, which was proposed by Tseytlin as an effective action on several D-branes, and its noncommutative counterpart for slowly varying fields. This confirms the equivalence between the two descriptions of the D-branes using an ordinary gauge theory with a constant B field background and a noncommutative gauge theory, claimed by Seiberg and Witten. We also construct the general forms of the $2n$-derivative terms for non-Abelian gauge fields which are consistent with the equivalence in the approximation of neglecting $(2n+2)$-derivative terms.

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1 Introduction

It has been known that the effective theory on D-branes in a background B field has two descriptions which are an ordinary gauge theory and a noncommutative gauge theory [1] [2]. To relate these, Seiberg and Witten proposed that the noncommutative gauge theory is indeed equivalent to the ordinary gauge theory by a field redefinition, called Seiberg-Witten map [3].

In the case of a D-brane, it has been known that the effective action on the brane is Born-Infeld action[†]if all derivative terms are neglected [4]-[6].[‡] Thus the Born-Infeld action should be consistent with the equivalence in this approximation. In fact this was shown in [3] by constructing a family of actions parameterized by a parameter of noncommutativity, θ. The family of actions contain the ordinary Born-Infeld action with a constant B background and its noncommutative counterpart without it and it was shown that the family of the actions is θ-independent in the approximation. Moreover, in [8] [9] it has been shown that the D-brane action computed in the superstring theory is consistent with the equivalence up to two derivative terms.

The effective action of several D-branes is very important in the recent development of the understanding of the non-perturbative superstring theory, such as Matrix theory [10] and AdS/CFT correspondence [11]. Although, Tseytlin proposed [12] that if all derivative terms are neglected, the effective action on the branes is a non-Abelian generalization of the Born-Infeld action using the symmetrized trace over the Chan-Paton indices, the effective action of the D-branes has not been understood completely. Thus it may be important to establish the equivalence between the noncommutative and the ordinary descriptions in this non-Abelian case because the equivalence may provide a tool to derive the effective action of the D-branes.

In this paper we explicitly show the equivalence between the non-Abelian Born-Infeld action and its noncommutative counterpart in the approximation of neglecting derivative terms, using the differential equation which (partially) defines the Seiberg-Witten map.

[†] More precisely, an effective action on a D-brane in the approximation becomes Dirac-Born-Infeld action [7]. However the part of the action, which is independent of the adjoint scalars, has the same form as the Born-Infeld action. Since only this part will be used in this paper, we will not distinguish the two actions.

[‡] By derivative terms, we mean terms with n-derivatives acting on field strengths (not on gauge fields).

This is regarded as a non-trivial test of the equivalence for the non-Abelian case. To show this, it is important to keep the ordering of the field strengths which are $N \times N$ matrices, where N is a number of the D-branes. From this, the expansion with respect to a noncommutative parameter θ is not relevant for this case. Then, we compute a difference between an action parameterized by θ and one by $\theta + \delta\theta$ exactly and then show that it contains at least a derivative term. This implies that the noncommutative action is equivalent to the ordinary action in the approximation. We also construct general forms of the $2n$-derivative terms for non-Abelian gauge fields which are consistent with the equivalence in the approximation of neglecting $(2n+2)$-derivative terms as in the abelian case [8].

We note that it has been shown in [13] [9] that for the case of a D-brane in the bosonic string, we should modify the field redefinition by gauge-invariant but B-dependent correction terms involving metric to match the known two-derivative terms [14] [15], thus we should modify the differential equation also. It is reasonable to take into account the possibility of this type of modification, however, such modification is not expected to change the result obtained in this paper in the approximation of neglecting derivative terms since the modification may include the derivative term as shown in [13]. This problem will be discussed in detail in section 4.

This paper is organized as follows. In section 2, we briefly review the equivalence between noncommutative and ordinary gauge theories shown in [3]. In section 3, we show the equivalence between the non-Abelian Born-Infeld action and its noncommutative counterpart for slowly varying fields using the differential equation which relates the ordinary and noncommutative gauge fields. In section 4, we show that the ambiguity in the Seiberg-Witten map can be ignored to prove the equivalence. In section 5, we also construct general forms of the $2n$-derivative terms for non-Abelian gauge fields which are consistent with the equivalence in the approximation of neglecting $(2n+2)$-derivative terms. Finally section 6 is devoted to conclusion.

2 Noncommutative Gauge Theory

In this section we review the equivalence between noncommutative and ordinary gauge theories discussed in [3]. We consider open strings in flat space, with metric g_{ij}, in the presence of a constant B_{ij} and with a Dp-brane. Here we assume that B_{ij} has rank $p+1$ and $B_{ij} \neq 0$ only for $i, j = 1, \ldots, p+1$. The world-sheet action is

$$S = \frac{1}{4\pi\alpha'} \int_{\Sigma} g_{ij} \partial_a x^i \partial^a x^j - \frac{i}{2} \int_{\partial\Sigma} B_{ij} x^i \partial_\tau x^j - i \int_{\partial\Sigma} A_i(x) \partial_\tau x^i, \tag{2.1}$$

where Σ is the string world-sheet, ∂_τ is the tangential derivative along the world-sheet boundary $\partial\Sigma$ and A_i is a background gauge field. In the case that Σ is the upper half plane parameterized by $-\infty \leq \tau \leq \infty$ and $0 \leq \sigma \leq \infty$, the propagator evaluated at boundary points is [4]-[6]

$$\langle x^i(\tau) x^j(\tau') \rangle = -\alpha' (G^{-1})^{ij} \log(\tau - \tau')^2 + \frac{i}{2} \theta^{ij} \epsilon(\tau - \tau'), \tag{2.2}$$

where G and θ are the symmetric and antisymmetric tensors defined by

$$(G^{-1})^{ij} + \frac{1}{2\pi\alpha'} \theta^{ij} = \left(\frac{1}{g + 2\pi\alpha' B} \right)^{ij}. \tag{2.3}$$

In the case of N D-branes, we must consider the Chan-Paton factors and A_i and F_{ij} become $N \times N$ matrices. From considerations of the string S-matrix, the B dependence of the effective action for fixed G can be obtained by replacing ordinary multiplication in the effective action for $B = 0$ by the $*$ product defined by the formula

$$f(x) * g(x) = e^{\frac{i}{2}\theta^{ij} \frac{\partial}{\partial \xi^i} \frac{\partial}{\partial \zeta^j}} f(x+\xi) g(x+\zeta) \Big|_{\xi=\zeta=0}. \tag{2.4}$$

Using the point splitting reguralization, the effective action is invariant under a noncommutative gauge transformation

$$\hat{\delta} \hat{A}_i = \hat{D}_i \lambda, \tag{2.5}$$

where covariant derivative $\hat{D}_i$ is defined as

$$\hat{D}_i E(x) = \partial_i E(x) + i \left(E(x) * \hat{A}_i - \hat{A}_i * E(x) \right). \tag{2.6}$$

On the other hand, using Pauli-Villars regularization, S is invariant under ordinary gauge transformation

$$\delta_o A_i = \partial_i \lambda. \tag{2.7}$$

Therefore, the effective Lagrangian obtained in this way becomes ordinary gauge theory. Therefore this ordinary gauge theory and the corresponding noncommutative gauge theory are equivalent under the field redefinition $\hat{A} = \hat{A}(A)$. Because the two different gauge invariance should satisfy $\hat{A}(A)+\hat{\delta}_{\hat{\lambda}}\hat{A}(A) = \hat{A}(A+\delta_\lambda A)$, the mapping of A to $\hat{A}$ is obtained as a differential equation for θ,

$$\begin{aligned} \delta\hat{A}_i(\theta) = \delta\theta^{kl}\frac{\partial}{\partial\theta^{kl}}\hat{A}_i(\theta) &= -\frac{1}{4}\delta\theta^{kl}\left[\hat{A}_k * (\partial_l\hat{A}_i + \hat{F}_{li}) + (\partial_l\hat{A}_i + \hat{F}_{li}) * \hat{A}_k\right] \\ \delta\hat{F}_{ij}(\theta) = \delta\theta^{kl}\frac{\partial}{\partial\theta^{kl}}\hat{F}_{ij}(\theta) &= \frac{1}{4}\delta\theta^{kl}[2\hat{F}_{ik} * \hat{F}_{jl} + 2\hat{F}_{jl} * \hat{F}_{ik} \\ &\quad -\hat{A}_k * \left(\hat{D}_l\hat{F}_{ij} + \partial_l\hat{F}_{ij}\right) - \left(\hat{D}_l\hat{F}_{ij} + \partial_l\hat{F}_{ij}\right) * \hat{A}_k], \end{aligned} \tag{2.8}$$

where

$$\hat{F}_{ij} = \partial_i\hat{A}_j - \partial_j\hat{A}_i - i\hat{A}_i * \hat{A}_j + i\hat{A}_j * \hat{A}_i. \tag{2.9}$$

Furthermore, in [3] it has been proposed that the effective action can be written for an arbitrary values of θ. More precisely for given physical parameters g_s, g_{ij} and B_{ij} and an auxiliary parameter θ, we define G_s, G_{ij} and a two form Φ_{ij} as

$$\begin{aligned} \left(\frac{1}{G+2\pi\alpha'\Phi}\right)^{ij} &= -\frac{1}{2\pi\alpha'}\theta^{ij} + \left(\frac{1}{g+2\pi\alpha' B}\right)^{ij} \\ G_s &= g_s\left(\det\left(-\frac{1}{2\pi\alpha'}\theta + \frac{1}{g+2\pi\alpha' B}\right)\det(g+2\pi\alpha' B)\right)^{-\frac{1}{2}}. \end{aligned} \tag{2.10}$$

Then the effective action $\hat{S}(G_s, G, \Phi, \theta;\, \hat{F})$, in which the multiplication is the θ-dependent $*$ product, is actually θ-independent, i.e. $\hat{S}(G_s, G, \Phi, \theta;\, \hat{F}) = S(g_s, g, B, \theta = 0;\, F)$. The effective action including Φ may be obtained using a regularization which interpolates between Pauli-Villars and point splitting as in [16]. In this paper, we simply assume this proposal.

In the rest of this section, we consider a single D-brane. In the approximation of neglecting the derivative terms, the effective Lagrangian is the Dirac-Born-Infeld Lagrangian

$$\mathcal{L}_{DBI} = \frac{1}{g_s(2\pi)^p(\alpha')^{\frac{p+1}{2}}}\sqrt{\det(g+2\pi\alpha'(B+F))}, \tag{2.11}$$

where $F_{ij} = \partial_i A_j - \partial_j A_i$. Here g_s is the closed string coupling and the normalization of the Lagrangian is same as the one taken in [3]. Therefore the equivalent noncommutative

gauge theory in the approximation has the following Lagrangian

$$\hat{\mathcal{L}}_{DBI} = \frac{1}{G_s(2\pi)^p(\alpha')^{\frac{p+1}{2}}}\sqrt{\det(G + 2\pi\alpha'\hat{F})}. \tag{2.12}$$

Note that all the multiplication entering the r.h.s of (2.12) can be regarded as the ordinary multiplication except those in the definition of $\hat{F}$ because of the approximation. From the requirement $\mathcal{L}_{DBI} = \hat{\mathcal{L}}_{DBI}$ for $F = 0$, the overall normalization G_s should be fixed as $G_s = g_s\sqrt{\det(G)/\det(g + 2\pi\alpha' B)}$.

In the approximation of neglecting the derivative of F, the equation

$$\delta\mathcal{L}_\Phi = \delta\theta^{kl}\frac{\partial\mathcal{L}_\Phi}{\partial\theta^{kl}}\bigg|_{g_s,g,B,A_i\,fixed} = \text{total derivative} + \mathcal{O}(\partial^2), \tag{2.13}$$

should hold, where

$$\delta = \delta\theta^{kl}\frac{\partial}{\partial\theta^{kl}}. \tag{2.14}$$

Here $\mathcal{L}_\Phi$ is the Lagrangian defined as

$$\mathcal{L}_\Phi = \frac{1}{G_s(2\pi)^p(\alpha')^{\frac{p+1}{2}}}\sqrt{\det(G + 2\pi\alpha'(\hat{F} + \Phi))}, \tag{2.15}$$

where the multiplication is the $*$ product except in the definition of $\hat{F}$. Below for simplicity we set $2\pi\alpha' = 1$. The variation of G_s, G and Φ are

$$\begin{aligned} \delta G_s &= \frac{1}{2}G_s\mathrm{Tr}(\Phi\delta\theta), \\ \delta G &= G\delta\theta\Phi + \Phi\delta\theta G, \\ \delta\Phi &= \Phi\delta\theta\Phi + G\delta\theta G, \end{aligned} \tag{2.16}$$

and the variation of $\hat{F}$ is

$$\begin{aligned} \delta\hat{F}_{ij} &= -(\hat{F}\delta\theta\hat{F})_{ij} - \hat{A}_k\delta\theta^{kl}\frac{1}{2}(\partial_l + \hat{D}_l)\hat{F}_{ij} + \mathcal{O}(\partial^4) \\ &= -(\hat{F}\delta\theta\hat{F})_{ij} - \hat{A}_k\delta\theta^{kl}(\partial_l - \frac{1}{2}\theta^{mn}\partial_n\hat{A}_l\partial_m)\hat{F}_{ij} + \mathcal{O}(\partial^4). \end{aligned} \tag{2.17}$$

Following [3], we get

$$\begin{aligned} &\delta\left(\frac{1}{G_s}\det(G + \hat{F} + \Phi)^{\frac{1}{2}}\right) \\ &= -\frac{1}{2}\frac{1}{G_s}\det(G + \hat{F} + \Phi)^{\frac{1}{2}}\left(\mathrm{Tr}(\hat{F}\delta\theta) + \left(\frac{1}{G + \hat{F} + \Phi}\right)_{ji}\hat{A}_k\delta\theta^{kl}\frac{1}{2}(\partial_l + \hat{D}_l)\hat{F}_{ij}\right), \end{aligned} \tag{2.18}$$

where the multiplication is the ordinary one except in $\hat{F}$ and $\hat{D}_l$. Now using

$$\frac{1}{2}(\partial_l + \hat{D}_l)\hat{A}_k - \frac{1}{2}(\partial_k + \hat{D}_k)\hat{A}_l = \hat{D}_l\hat{A}_k - \partial_k\hat{A}_l = \hat{F}_{lk}, \tag{2.19}$$

we see that

$$\begin{aligned} &\delta\theta^{kl}(\partial_l + \hat{D}_l)\left(\hat{A}_k \det(G + \hat{F} + \Phi)^{\frac{1}{2}}\right) \\ &= \delta\theta^{kl} \det(G + \hat{F} + \Phi)^{\frac{1}{2}} \left(\hat{F}_{lk} + \frac{1}{2}\left(\frac{1}{G + \hat{F} + \Phi}\right)_{ji} \hat{A}_k(\partial_l + \hat{D}_l)\hat{F}_{ij}\right) + \mathcal{O}(\partial^4), \end{aligned} \tag{2.20}$$

is a total derivative. Thus we obtain the desired result

$$\delta\left(\frac{1}{G_s}\det(G + \hat{F} + \Phi)^{\frac{1}{2}}\right) = \text{total derivative} + \mathcal{O}(\partial^4). \tag{2.21}$$

3 The Equivalence between non-Abelian Born-Infeld Actions

In this section, we consider the non-Abelian ($N \times N$ matrix valued) gauge field A_i on the N D-branes. In this case, we should keep the ordering of F_{ij} in the action because of the non-Abelian nature of F_{ij} even for $B = 0$. However, we will see that a noncommutative extension of the non-Abelian Born-Infeld action satisfies the equivalence in the approximation of neglecting derivative terms.

Let us consider the non-Abelian Born-Infeld action proposed by Tseytlin [12]

$$\mathcal{L}_{NBI} = c_p \operatorname{Str}_{\{F_{ij}\}} \sqrt{\det(g_{ij} + (B + F)_{ij})}, \tag{3.1}$$

where the determinant is computed with respect to the worldvolume indices i, j only and $\operatorname{Str}_{\{F_{ij}\}}$ means to symmetrize with respect to F_{ij} and to take trace over the Chan-Paton indices,

$$\operatorname{Str}_{\{F_{ij}\}}(F_{i_1j_1}\cdots F_{i_nj_n}) \equiv \operatorname{tr}\left(\operatorname{Sym}_{\{F_{ij}\}}(F_{i_1j_1}\cdots F_{i_nj_n})\right), \tag{3.2}$$

where

$$\operatorname{Sym}_{\{F_{ij}\}}(F_{i_1j_1}\cdots F_{i_nj_n}) \equiv \frac{1}{n!}(F_{i_1j_1}\cdots F_{i_nj_n} + \text{all permutations}), \tag{3.3}$$

and ${c_p}^{-1} = g_s(2\pi)^p(\alpha')^{\frac{p+1}{2}} = g_s(2\pi)^{\frac{p-1}{2}}$. Here det and square root should be taken as if F_{ij} is not the $N \times N$ matrix but a number and tr is the trace over the $N \times N$ Chan-Paton

indices. In other words, we should forget the ordering of F_{ij}. Note that the ambiguity of the ordering is fixed by the symmetrization. An explicit form of $\mathcal{L}_{NBI}$ is

$$\mathcal{L}_{NBI} = c_p \,\mathrm{tr}\left[1 + \frac{1}{4}F_{ij}F_{ij} - \frac{1}{96}\left(8F_{ij}F_{kj}F_{il}F_{kl} + 4F_{ij}F_{kj}F_{kl}F_{il} \right.\right. \\ \left.\left. -2F_{ij}F_{ij}F_{kl}F_{kl} - F_{ij}F_{kl}F_{ij}F_{kl}\right) + \cdots\right], \quad (3.4)$$

where we set $g_{ij} = \delta_{ij}$ for notational simplicity.

In [12], it was argued that this non-Abelian Born-Infeld action becomes the effective action on the N D-branes when we neglect the covariant derivative terms $D_k F_{ij}$. Here we should treat the commutator term $[F_{kl}, F_{ij}]$ as a derivative term since $[D_k, D_l]F_{ij} = -i[F_{kl}, F_{ij}]$.

A noncommutative extension of the non-Abelian Born-Infeld action $\mathcal{L}_{NBI*}$ is the Lagrangian defined as

$$\mathcal{L}_{NBI*} = \mathrm{Str}_{\{\hat{F}_{ij}\}}\left(\mathcal{L}_\Phi\right), \quad (3.5)$$

where

$$\mathcal{L}_\Phi = \frac{1}{G_s}\left[\det{}_*(G + \hat{F} + \Phi)\right]_*^{\frac{1}{2}} = \frac{\det G^{\frac{1}{2}}}{G_s}\left(\det{}_*(1 + G^{-1}(\hat{F} + \Phi))\right)_*^{\frac{1}{2}}, \quad (3.6)$$

The multiplication in (3.6) is the $*$ product as indicated by $(\det_*(\cdots))_*^{\frac{1}{2}}$. We will not explicitly indicate the $*$ product below since we will always use the $*$ product. Note that the $*$ product is reduced to the ordinary product for constant fields.

In this noncommutative case, we will regard $\hat{D}_k\hat{F}_{ij}$ as a derivative term for an arbitrary θ. In order to see that this is natural, we will prove a claim that if $D_k F_{ij} = 0$ for any i, j, k at $\theta = 0$, then $\hat{D}_k\hat{F}_{ij} = 0$ for any θ and any i, j, k. The derivation of $\hat{D}\hat{F}$ with respect to θ can be computed as

$$\delta\left(\hat{D}_k\hat{F}_{ij}\right) = \frac{1}{4}\delta\theta^{pq}\left(2\hat{D}_k\{\hat{F}_{ip}, \hat{F}_{jq}\} + 2\{\hat{D}_q\hat{F}_{ij}, \hat{F}_{pk}\} - \{\hat{A}_p, (\hat{D}_q + \partial_q)\hat{D}_k\hat{F}_{ij}\}\right). \quad (3.7)$$

The r.h.s. of (3.7) vanishes if $\hat{D}_k\hat{F}_{ij} = 0$, which implies that the above claim is true.

Let us compute the variation with respect to $\mathcal{L}_{NBI*}$ with respect to θ

$$\delta\mathcal{L}_{NBI*} = \delta\theta^{kl}\frac{\partial\mathcal{L}_{NBI*}}{\partial\theta^{kl}}\bigg|_{g_s, g, B, A_i\, fixed} = \Delta_\Phi + \Delta_*, \quad (3.8)$$

where g_s, g_{ij}, B_{ij} and A_i are fixed. Here the term Δ_Φ includes the contributions from δG_s, δG_{ij}, $\delta\Phi_{ij}$ and $\delta\hat{F}_{ij}$ and the term Δ_* includes the contributions from the variation of θ in the $*$ product.

Using the property of the Str,

$$\delta\left(\mathrm{Str}_{\{\hat{F}_{ij}\}}\left(\mathcal{L}_\Phi(\hat{F}_{ij})\right)\right) = \mathrm{Str}_{\{\hat{F}_{ij},\delta\hat{F}_{ij}\}}\left(\sum_{k,l}\delta\hat{F}_{kl}\frac{\partial}{\partial\hat{F}_{kl}}\mathcal{L}_\Phi(\hat{F}_{ij})\right) + \cdots, \tag{3.9}$$

where the ellipsis denotes the contributions from the variation of θ in the $*$ product corresponding to Δ_*, we can find

$$\begin{aligned}\Delta_\Phi &= \frac{1}{2}\mathrm{Str}_{\{\hat{F}_{ij},\delta\hat{F}_{ij}\}}\left[\mathcal{L}_\Phi\,\mathrm{Tr}\left(\delta\theta\Phi + \frac{1}{1+G^{-1}(\hat{F}+\Phi)}\left(\delta G^{-1}(\hat{F}+\Phi)+G^{-1}(\delta\hat{F}+\delta\Phi)\right)\right)\right] \\ &= \frac{1}{2}\mathrm{Str}_{\{\hat{F}_{ij},\delta\hat{F}_{ij}\}}\left[\mathcal{L}_\Phi\mathrm{Tr}\left(\frac{1}{1+G^{-1}(\hat{F}+\Phi)}\left(\delta\theta(G+\Phi)-G^{-1}\Phi\delta\theta\hat{F}+G^{-1}\delta\hat{F}\right)\right)\right].\end{aligned} \tag{3.10}$$

Here Tr is trace over worldvolume indices and $\delta\hat{F}$ is evaluated after taking the symmetrized trace. Note that we can insert the $N\times N$ identity matrix $\{1\}_{ij} = \{\frac{1}{1+G^{-1}(\hat{F}+\Phi)}(1+G^{-1}(\hat{F}+\Phi))\}_{ij}$ into the $\mathrm{Str}(\mathrm{Tr}(\cdots))$ without changing the result. Substituting $\delta\theta(G+\Phi) = \delta\theta G(1+G^{-1}(\hat{F}+\Phi)) - \delta\theta\hat{F}$ and $\mathrm{Tr}(\delta\theta G) = 0$ into Δ_Φ, we see

$$\begin{aligned}\Delta_\Phi &= \frac{1}{2}\mathrm{Str}_{\{\hat{F}_{ij},\delta\hat{F}_{ij}\}}\left[\mathcal{L}_\Phi\,\mathrm{Tr}\left(\frac{1}{1+G^{-1}(\hat{F}+\Phi)}\left(-\delta\theta\hat{F}-G^{-1}\Phi\delta\theta\hat{F}+G^{-1}\delta\hat{F}\right)\right)\right] \\ &= \frac{1}{2}\mathrm{Str}_{\{\hat{F}_{ij},\delta\hat{F}_{ij}\}}\left[\mathcal{L}_\Phi\,\mathrm{Tr}\left(\frac{1}{1+G^{-1}(\hat{F}+\Phi)}G^{-1}\left(\hat{F}\delta\theta\hat{F}+\delta\hat{F}\right)-\delta\theta\hat{F}\right)\right].\end{aligned} \tag{3.11}$$

As in the abelian case [3], we consider to add a total derivative term

$$\begin{aligned}\Delta_{t.d.} &\equiv \frac{1}{2}\delta\theta^{kl}\mathrm{tr}\left((\partial_l+\hat{D}_l)\,\mathrm{Sym}_{\{\hat{A}_i,\hat{F}_{ij}\}}\left(\hat{A}_k\mathcal{L}_\Phi\right)\right) \\ &= \frac{1}{2}\mathrm{Str}_{\{\hat{A}_i,\hat{F}_{ij},(\partial_l+D_l)\hat{F}_{ji}\}}\Bigg[\mathcal{L}_\Phi\Bigg(\frac{1}{2}\left(\frac{1}{1+G^{-1}(\hat{F}+\Phi)}G^{-1}\right)^{ij}\delta\theta_{kl}\hat{A}_k(\partial_l+\hat{D}_l)\hat{F}_{ji} \\ &\qquad +\mathrm{Tr}(\delta\theta\hat{F})\Bigg)\Bigg],\end{aligned} \tag{3.12}$$

to $\delta\mathcal{L}_{NBI*}$. Here we have used (2.19) which is valid for the non-Abelian fields. Let us define differential operators δ' and $\tilde{\delta}$ as

$$\begin{aligned}\delta'\hat{F}_{ij} &\equiv -\frac{1}{2}\left[(\hat{F}\delta\theta\hat{F})_{ij}-(\hat{F}\delta\theta\hat{F})_{ji}\right], \\ \delta' G_s &= \delta' G_{ij} = \delta'\Phi_{ij} = \delta'\theta = 0,\end{aligned} \tag{3.13}$$

and

$$\tilde{\delta}E = \frac{1}{4}\delta\theta^{kl}\left\{\hat{A}_k\,,\,(\partial_l+\hat{D}_l)E\right\}, \tag{3.14}$$

where E is an arbitrary function of $\hat{A}_i$ and $\{\,,\,\}$ is the anticommutator. The operator δ', which is supposed to satisfy Leibniz rule, should act only a function of $\hat{F}$ which contains neither $\hat{A}_i$ nor $\hat{D}_l$ explicitly. Note that $(\delta' - \tilde{\delta})$ is not equivalent to δ though $(\delta' - \tilde{\delta})\hat{F}_{ij} = \delta\hat{F}_{ij}$. Then substituting $\delta\hat{F}_{ij} = (\delta' - \tilde{\delta})\hat{F}_{ij}$ into $\delta\hat{F}$ in (3.11), we can see

$$
\begin{aligned}
\Delta_\Phi + \Delta_{t.d.} &= \frac{1}{2}\mathrm{Str}_{\{\hat{F},\delta\hat{F},\hat{A},(\partial+\hat{D})\hat{F}\}}\Bigg[\mathcal{L}_\Phi\left(\frac{1}{1+G^{-1}(\hat{F}+\Phi)}G^{-1}\right)^{ij} \\
&\qquad \times\left(\delta\hat{F}_{ij} + \frac{1}{2}(\hat{F}\delta\theta\hat{F})_{ij} + \frac{1}{2}(\hat{F}\delta\theta\hat{F})_{ji} + \frac{1}{4}\delta\theta^{kl}\left\{\hat{A}_k\,,\,(\partial_l+\hat{D}_l)\hat{F}_{ji}\right\}\right)\Bigg] \\
&= (\delta' - \tilde{\delta})\left(\mathrm{Str}_{\{\hat{F}_{ij}\}}\left[\mathcal{L}_\Phi\right]\right) - \left(\mathrm{Str}_{\{\hat{A}_i,\hat{F}_{ij},(\partial_l+\hat{D}_l)\hat{F}_{ji}\}}\left[(\delta'-\tilde{\delta})\mathcal{L}_\Phi\right]\right) \\
&= \delta'\left(\mathrm{Str}_{\{\hat{F}_{ij}\}}\left[\mathcal{L}_\Phi\right]\right) - \left(\mathrm{Str}_{\{\hat{F}_{ij}\}}\left[\delta'\mathcal{L}_\Phi\right]\right) \\
&\quad -\tilde{\delta}\left(\mathrm{Str}_{\{\hat{F}_{ij}\}}\left[\mathcal{L}_\Phi\right]\right) + \left(\mathrm{Str}_{\{\hat{A}_i,\hat{F}_{ij},(\partial_l+\hat{D}_l)\hat{F}_{ji}\}}\left[\tilde{\delta}\mathcal{L}_\Phi\right]\right).
\end{aligned}
\tag{3.15}
$$

In the above equation, $\delta'\left(\mathrm{Str}_{\{\hat{F}_{ij}\}}\left[\mathcal{L}_\Phi\right]\right) - \left(\mathrm{Str}_{\{\hat{F}_{ij}\}}\left[\delta'\mathcal{L}_\Phi\right]\right)$ would vanish if we neglect the ordering of $\hat{F}$. Thus this can be expressed as a sum of the polynomials of $\hat{F}$ which contain at least a commutator of $\hat{F}$'s and is considered to be derivative terms in the sense of [12]. However, the last line $-\tilde{\delta}\left(\mathrm{Str}_{\{\hat{F}_{ij}\}}\left[\mathcal{L}_\Phi\right]\right) + \left(\mathrm{Str}_{\{\hat{A}_i,\hat{F}_{ij},(\partial_l+\hat{D}_l)\hat{F}_{ji}\}}\left[\tilde{\delta}\mathcal{L}_\Phi\right]\right)$ is not considered to be derivative terms because it contains $\partial_l\hat{F}$ and $\hat{A}_i$. To proceed further, we expand this in $\hat{F}$ and concentrate on a term

$$
L = -\tilde{\delta}\left(\mathrm{Str}_{\{\hat{F}_{ij}\}}\left[\hat{F}_1\hat{F}_2\cdots\hat{F}_n\right]\right) + \left(\mathrm{Str}_{\{\hat{A}_i,\hat{F}_{ij},(\partial_l+\hat{D}_l)\hat{F}_{ji}\}}\left[\tilde{\delta}(\hat{F}_1\hat{F}_2\cdots\hat{F}_n)\right]\right), \tag{3.16}
$$

where $\hat{F}_l = \hat{F}_{i_l j_l}$. Moving $\hat{A}_i$ in the second term of (3.16) to the head of the term and extracting the terms which have the form $\hat{F}_1\hat{F}_2\cdots\hat{F}_n$ concerning the ordering of $\hat{F}$ from (3.16), we find

$$
\begin{aligned}
L &= -\frac{1}{4}\delta\theta^{kl}\mathrm{tr}\left[\sum_{p=1}^{n}\hat{F}_1\cdots\hat{F}_{p-1}(\hat{A}_k(\hat{D}_l+\partial_l)\hat{F}_p + ((\hat{D}_l+\partial_l)\hat{F}_p)\hat{A}_k)\,\hat{F}_{p+1}\cdots\hat{F}_n\right] \\
&\quad +\frac{1}{2}\delta\theta^{kl}\mathrm{tr}\left[\hat{A}_k\sum_{p=1}^{n}\hat{F}_1\cdots\hat{F}_{p-1}((\hat{D}_l+\partial_l)\hat{F}_p)\hat{F}_{p+1}\cdots\hat{F}_n\right] + \text{total derivative} \\
&= \frac{i}{4}\delta\theta^{kl}\mathrm{tr}\Bigg[\sum_{p=2}^{n}((\hat{D}_k-\partial_k)(\hat{F}_1\cdots\hat{F}_{p-1}))\,((\hat{D}_l+\partial_l)\hat{F}_p)\,\hat{F}_{p+1}\cdots\hat{F}_n \\
&\qquad -\sum_{p=1}^{n-1}\hat{F}_1\cdots\hat{F}_{p-1}((\hat{D}_l+\partial_l)\hat{F}_p)\,(\hat{D}_k-\partial_k)(\hat{F}_{p+1}\cdots\hat{F}_n)\Bigg] + \text{total derivative}
\end{aligned}
$$

$$
\begin{aligned}
= \ & \frac{i}{2}\delta\theta^{kl}\mathrm{tr}\sum_{p=2}^{n}\Big[\ (\hat{D}_k(\hat{F}_1\cdots\hat{F}_{p-1}))\,(\hat{D}_l\hat{F}_p)\,\hat{F}_{p+1}\cdots\hat{F}_n \\
& \qquad -(\partial_k(\hat{F}_1\cdots\hat{F}_{p-1}))\,(\partial_l\hat{F}_p)\,\hat{F}_{p+1}\cdots\hat{F}_n\ \Big] + \text{total derivative},
\end{aligned}
\tag{3.17}
$$

where we have used $[\hat{A}_k,\hat{F}] = i(\hat{D}_k-\partial_k)\hat{F}$. Note that the variation δ of the $*$ product can be read from

$$
\delta(X_1\cdots X_n) = \frac{i}{2}\delta\theta^{kl}\sum_{p=2}^{n}(\partial_k(X_1\cdots X^{p-1}))\,(\partial_l X^p)X^{p+1}\cdots X^n, \tag{3.18}
$$

where $\delta X_i = 0$. Then the second term in (3.17) is canceled by the contribution from Δ_*.

Therefore from (3.15), (3.17) and (3.18), we finally obtain that

$$
\begin{aligned}
\delta\mathcal{L}_{NBI*} &= \Delta_\Phi + \Delta_* - \Delta_{t.d.} + \text{total derivative} \\
&= \delta'\left(\mathrm{Str}_{\{\hat{F}_{ij}\}}\,[\mathcal{L}_\Phi]\right) - \left(\mathrm{Str}_{\{\hat{F}_{ij}\}}\,[\delta'\mathcal{L}_\Phi]\right) \\
&\qquad + \delta_{\hat{D}}\left(\mathrm{Str}_{\{\hat{F}_{ij}\}}\,[\mathcal{L}_\Phi]\right) - \text{total derivative},
\end{aligned}
\tag{3.19}
$$

where the linear operator $\delta_{\hat{D}}$ is defined as

$$
\delta_{\hat{D}}(\hat{F}_1\cdots\hat{F}_n) = \frac{i}{2}\delta\theta^{kl}\sum_{p=2}^{n}(\hat{D}_k(\hat{F}_1\cdots\hat{F}_{p-1}))\,(\hat{D}_l\hat{F}_p)\hat{F}_{p+1}\cdots\hat{F}_n. \tag{3.20}
$$

Since (3.19) does not contain $\hat{A}_i$ or $\partial_k\hat{F}_{ij}$, $\delta\mathcal{L}_{NBI*}$ is derivative terms in the sense of [12] plus total derivative terms. Therefore we conclude that the non-Abelian Born-Infeld action satisfies the equivalence in the approximation of neglecting derivative terms.

We note that (3.19) does not contains $\mathcal{O}(\hat{D}^2\hat{F}^2)$ terms since $\delta\theta^{kl}(\hat{D}_k\hat{F}_{ij})(\hat{D}_l\hat{F}_{ij}) = 0$ and $\delta'\left(\mathrm{Str}_{\{\hat{F}_{ij}\}}\left[\hat{F}_{ij}\hat{F}_{ij}\right]\right) - \left(\mathrm{Str}_{\{\hat{F}_{ij}\}}\left[\delta'(\hat{F}_{ij}\hat{F}_{ij})\right]\right) = 0$. However, (3.19) may contain $\mathcal{O}(\hat{D}^2\hat{F}^{2n})$ terms, where $n > 1$, then the non-Abelian Born-Infeld action itself does not satisfies the equivalence and some derivative corrections or/and modification of the Seiberg-Witten map (2.8) should be needed. It is an interesting problem to find such corrections and to compare those with the result obtained in [17].

4 Ambiguity in the Seiberg-Witten Map

Here we will shortly discuss the ambiguity in the Seiberg-Witten map and its effect on the above proof of the equivalence. When we regard (2.8) as the partial differential equation,

it is not integrable. Thus the solution of it depends on the path in the θ space although the path-dependence is absorbed by the gauge transformation and the field redefinition at fixed θ, as explicitly shown in [18]. In fact, by reconsidering the derivation of (2.8), we can see that (2.8) will not be imposed for all θ. The equation (2.8) should be imposed at each θ only modulo the gauge transformation and the field redefinition at fixed θ. Then, strictly speaking, (2.8) should not be regarded as the partial differential equation and there are ambiguities in the solution of the equation (2.8) modulo these.

However, the ambiguities are not relevant for the proof of the equivalence. The reason is the following. At first order of $\delta\theta$ the ambiguity arising from the gauge transformation discussed in [18] is

$$\begin{aligned}\delta\hat{F}_{ij}(\theta) &= \frac{1}{4}\delta\theta^{kl}\Big(2\hat{F}_{ik}*\hat{F}_{jl}+2\hat{F}_{jl}*\hat{F}_{ik}-\hat{A}_k*\Big(\hat{D}_l\hat{F}_{ij}+\partial_l\hat{F}_{ij}\Big)-\Big(\hat{D}_l\hat{F}_{ij}+\partial_l\hat{F}_{ij}\Big)*\hat{A}_k \\ &\quad -i\Big[\hat{F}_{ij},\alpha\hat{F}_{kl}+\beta[\hat{A}_k,\hat{A}_l\,]\Big]\Big).\end{aligned}\tag{4.1}$$

We can easily see that the last term does not contribute the $\delta\mathcal{L}_{NBI}$ since it has the form of the gauge transformation. The ambiguity arising from the field redefinition should have the form $\delta\hat{A}_i\sim\delta\theta^{kl}H_{ikl}(G,\theta,\Phi,\hat{F},\hat{D}\hat{F},\hat{D}\hat{D}\hat{F},\cdots)$ because of the gauge invariance. Note that the number of $\hat{D}$ in H_{ikl} is odd. Thus the contributions from this term to $\delta\hat{F}$ is $\delta\hat{F}_{ij}\ \sim\delta\theta^{kl}(\hat{D}_iH_{jkl}-\hat{D}_jH_{ikl})$ in the first order of $\delta\theta$. Therefore the corrections to $\delta\mathcal{L}_{NBI}$ from this term are derivative terms and we conclude that we can ignore the ambiguities to prove the equivalence in the approximation.

In order to prove the equivalence including the higher derivative terms, we should take into account th ambiguity. From the fact that the θ is not appear explicitly in (3.19) and δ(derivative corrections), we can find the possible form of the Seiberg-Witten map is

$$\begin{aligned}\delta\hat{F}_{ij}(\theta) &= \frac{1}{4}\delta\theta^{kl}\Big(2\hat{F}_{ik}*\hat{F}_{jl}+2\hat{F}_{jl}*\hat{F}_{ik}-\hat{A}_k*\Big(\hat{D}_l\hat{F}_{ij}+\partial_l\hat{F}_{ij}\Big)-\Big(\hat{D}_l\hat{F}_{ij}+\partial_l\hat{F}_{ij}\Big)*\hat{A}_k \\ &\quad -(\hat{D}_iH_{jkl}-\hat{D}_jH_{ikl})\Big),\end{aligned}\tag{4.2}$$

where

$$H_{ikl}=H_{ikl}(G^{-1},\hat{F}+\Phi,\hat{D}\hat{F},\hat{D}\hat{D}\hat{F},\cdots).\tag{4.3}$$

Note that the (4.3) depends on α' although it is not explicitly indicated.

5 Derivative Corrections

In this section, we consider the derivative terms which are consistent with the equivalence. The tree level effective action of the D-branes in the superstring theory is expected to be the non-Abelian Born-Infeld action with an appropriate linear combination of these derivative terms because the effective action should satisfy the equivalence. Thus it is desired to find the general forms of the derivative corrections which satisfy the equivalence. As a first step to find them, we will construct the $2m$-derivative terms which satisfy the equivalence in the approximation of neglecting $(2m+2)$-derivative terms.

The general forms of the derivative corrections which satisfy the equivalence in the approximation were obtained in [8] for the abelian case. Below we will generalize the result obtained in [8] to the non-Abelian case. To do this, we define

$$\begin{aligned}(h_S)^{ij} &= \left(\frac{1}{G+\hat{F}+\Phi}\right)^{ij}_{\text{sym}} = \frac{1}{2}\left(\frac{1}{G+\hat{F}+\Phi}\right)^{ij} + \frac{1}{2}\left(\frac{1}{G-\hat{F}-\Phi}\right)^{ij} \\ &= \left(\frac{1}{G+\hat{F}+\Phi}G\frac{1}{G-\hat{F}-\Phi}\right)^{ij}, \end{aligned} \tag{5.1}$$

and denote an arbitrary $(0,3m)$ tensor of a form

$$\left\{(\hat{D}\hat{F})\cdots(\hat{D}\hat{F})\right\}_{p_1p_2\cdots p_{3m}}, \tag{5.2}$$

by $J_{p_1p_2\cdots p_{3m}}$. For example, we can take $J_{p_1p_2\cdots p_6} = \hat{D}_{p_1}\hat{F}_{p_2p_3}\hat{D}_{p_4}\hat{F}_{p_5p_6}$ for $m=2$. This tensor will be used only in Str or Sym, then the ordering of $\hat{D}\hat{F}$ in it will fixed.

Now we consider the m-derivative terms

$$\mathcal{L}_m = \text{Str}_{\{\hat{F}_{ij},\hat{D}_k\hat{F}_{ij}\}}\left(\mathcal{L}_\Phi\, L_m\right), \tag{5.3}$$

where

$$L_m = (h_S)^{p_1p_2}(h_S)^{p_3p_4}\cdots(h_S)^{p_{3m-1}p_{3m}}\; J_{p_1p_2\cdots p_{3m}}, \tag{5.4}$$

is a m-derivative terms and $\mathcal{L}_\Phi$ is the θ-dependent non-Abelian Born-Infeld Lagrangian defined in (3.6). We separate the variation of $\mathcal{L}_m$ with respect to θ to three parts such that

$$\delta\mathcal{L}_m = \Delta^m_\Phi + \Delta^m_L + \Delta^m_*, \tag{5.5}$$

where Δ^m_Φ and Δ^m_L are the contributions from $\delta G_s, \delta G_{ij}, \delta\Phi_{ij}, \delta\hat{F}_{ij}$ and $\delta(\hat{D}_k\hat{F}_{ij})$ in $\mathcal{L}_\Phi$ and in L_m, respectively, and Δ^m_* comes from the variation of θ in the $*$ product except in $\hat{F}$ and $\hat{D}\hat{F}$.

Next we will compute $(\delta+\tilde{\delta})(\hat{D}_k\hat{F})$. We can show that

$$\begin{aligned}[][\delta, \hat{D}_k]E &= -\frac{i}{4}\theta^{pq}\left(\left\{[E,\hat{A}_p],(\partial_q\hat{A}_k+\hat{F}_{qk})\right\}+\{\hat{A}_p,[E,\partial_q\hat{A}_k+\hat{F}_{qk}]\}\right) \\ &\quad -\frac{1}{2}\delta\theta^{pq}(\partial_p E\partial_q\hat{A}_k-\partial_p\hat{A}_k\partial_q E),\end{aligned} \tag{5.6}$$

where $\hat{D}_kE=\partial_kE+i[E,\hat{A}_k]$, and

$$[\tilde{\delta},\hat{D}_k]E=\frac{1}{4}\theta^{pq}\left(-\left\{\hat{D}_k\hat{A}_p,(\partial_q+\hat{D}_q)E\right\}+i\left\{\hat{A}_p,[E,\hat{F}_{qk}+\partial_q\hat{A}_k]\right\}\right). \tag{5.7}$$

From these, after some computations we find a simple result

$$[\delta+\tilde{\delta},\hat{D}_k]E=-\frac{1}{2}\delta\theta^{pq}\{\hat{F}_{kp},\hat{D}_qE\}. \tag{5.8}$$

Then using

$$(\delta+\tilde{\delta})\hat{F}_{ij}=-\frac{1}{2}\delta\theta^{pq}\{\hat{F}_{ip},\hat{F}_{qj}\}, \tag{5.9}$$

we obtain

$$\begin{aligned}(\delta+\tilde{\delta})(\hat{D}_k\hat{F}_{ij}) &= -\frac{1}{2}\delta\theta^{pq}\left(\{\hat{D}_k\hat{F}_{ip},\hat{F}_{qj}\}+\{\hat{F}_{ip},\hat{D}_k\hat{F}_{qj}\}+\{\hat{F}_{kp},\hat{D}_q\hat{F}_{ij}\}\right) \\ &= -\frac{1}{2}\left(\hat{D}_k\{\hat{F}\delta\theta,\hat{F}\}_{ij}+\{(\hat{F}\delta\theta)_k{}^q,\hat{D}_q\hat{F}_{ij}\}\right).\end{aligned} \tag{5.10}$$

As in the abelian case [8], we also show that

$$(\delta+\tilde{\delta})(h_S)^{ij}=\left(h_S(\hat{F}\delta\theta)+(\delta\theta\hat{F})h_S\right)^{ij}+\delta_*(h_S)^{ij}+\cdots, \tag{5.11}$$

and then

$$(\delta+\tilde{\delta})L_m=\delta_*L_m+\cdots, \tag{5.12}$$

where $\delta_*(h_S)^{ij}$ and δ_*L_m are contributions coming from the variation of θ in the $*$ product and the ellipsis denotes terms involving a commutator $\hat{F}$, which are regarded as the $(m+2)$-derivative terms.

According to the discussion in the previous section, we finally find

$$\delta\mathcal{L}_m+\Delta^m_{t.d}=\Delta_{m+2}+\text{total derivative}, \tag{5.13}$$

where Δ_{m+2} is a $(m+2)$-derivative term and

$$\Delta^m_{t.d} \equiv \frac{1}{2}\delta\theta^{kl}\mathrm{tr}\left((\partial_l + \hat{D}_l)\,\mathrm{Sym}_{\{\hat{A},\hat{F},\hat{D}\hat{F}\}}\left(\hat{A}_k\mathcal{L}_\Phi\, L_m\right)\right), \tag{5.14}$$

is a total derivative term. Therefore the m-derivative correction (5.3) satisfies the equivalence in the approximation of neglecting $(m+2)$-derivative terms.

In [8], it was shown that a type of derivative corrections containing h_A, which is defined as

$$\begin{aligned}(h_A)^{ij} &= \left(\frac{1}{G+\hat{F}+\Phi}\right)^{ij}_{\mathrm{antisym}} = \frac{1}{2}\left(\frac{1}{G+\hat{F}+\Phi}\right)^{ij} - \frac{1}{2}\left(\frac{1}{G-\hat{F}-\Phi}\right)^{ij} \\ &= -\left(\frac{1}{G+\hat{F}+\Phi}(\hat{F}+\Phi)\frac{1}{G-\hat{F}-\Phi}\right)^{ij},\end{aligned} \tag{5.15}$$

also satisfies the equivalence. As in the above discussion on the derivative correction containing h_S, we can easily shown that the generalization of this type of derivative corrections to the non-Abelian gauge fields also satisfies the equivalence.

6 Conclusion

We have shown that the non-Abelian Born-Infeld action is equivalent to its noncommutative counterpart in the approximation of neglecting derivative terms not expanding the action with respect to the noncommutative parameter θ. We have also constructed the general forms of the $2n$-derivative terms for the non-Abelian gauge fields which are consistent with the equivalence in the approximation of neglecting $(2n+2)$-derivative terms. It may capture some general structures of the effective action of the D-branes.

It is interesting to generalize the results obtained in this paper to construct the action which satisfies the equivalence without the approximation of neglecting derivative terms, which may has applications, especially, for a relation between the nonlinear instanton [3] [19] [20] and the noncommutative instanton [21]. Since we should treat the non-Abelian gauge fields, there is the ordering problem even for the ordinary gauge fields, which has not been solved yet. Thus the constraints using the equivalence are expected to be important for determination of the effective action on the several D-branes. If we success to construct such an action, we would solve the ordering problem also.

To supplement this approach to obtaining the effective action of D-branes, it would be important to consider the supersymmetric extension of the action. The superfields in noncommutative geometry has been discussed in [22]-[24] and the supersymmetric non-Abelian Dirac-Born-Infeld action in noncommutative geometry was discussed in [25]. Then it might be interesting to consider supersymmetric noncommutative gauge theories and their equivalence relations.

The simplified Seiberg-Witten map [26] [27] may be also useful to construct the action consistent with the equivalence as in [28]. Although the simplified Seiberg-Witten map is different from the Seiberg-Witten map in the higher order of θ, the derivative corrections obtained in [8] using the Seiberg-Witten map coincide with those obtained in [28]. In order to proceed this method further, it is important to study the relation between the two maps.

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Note added:

While preparing this article for publication, we received the preprint [29] which discussed the general structure of the non-Abelian Born-Infeld action from the equivalence between ordinary and noncommutative gauge theories using an algebraic method. In particular, in two dimension the non-Abelian Born-Infeld action was recovered from the equivalence and its lowest derivative correction was found. On the other hand, the result obtained in this paper does not depend on the dimension of the D-branes.

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